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and for the angular motion, taking moments about the axis of the roll,  $\theta$  being the angular motion corresponding to  $x$ , and so

$$x = r\theta \dots (4), \quad \frac{d}{dt}(\rho \cdot \pi r^2 \cdot k^2 \dot{\theta}) = rF. \quad (5)$$

We have  $k^2 = r^2/2$  (6), and from (4),  $\dot{\theta} = \dot{x}/r$  (7). Substituting (6) and (7) in (5) and eliminating  $F$  from the resulting equation and (3), developing the derivatives and reducing,

$$\ddot{x} + \frac{7}{3} \cdot \frac{\dot{r}}{r} \dot{x} = \frac{2g \sin \phi}{3}. \quad (8)$$

Now, from (2),  $\dot{x} = -(2\pi r/a)\dot{r}$  (9),  $\ddot{x} = -(2\pi/a)(\dot{r}^2 + r\ddot{r})$  (10), and substituting these in (8) and arranging,

$$\ddot{r} + \frac{10}{3r} \dot{r}^2 = -\frac{ag \sin \phi}{3\pi r}. \quad (11)$$

Putting  $\dot{r}^2 = y$  (12), (11) is thrown into the form  $(dy/dr) + Py = Q$  (13), in which  $P = 20/3r$  (14), and  $Q = -(2ag \sin \phi/3\pi r)$  (15), the integral (13) being

$$y = e^{-\int P dr} \int e^{\int P dr} Q + Ce^{-\int P dr}. \quad (16)$$

Substituting (14) and (15) in (16) and performing the operations indicated,

$$y = -\frac{2ag \sin \phi}{3\pi} r^{-20/3} \cdot \frac{3}{20} r^{20/3} + Cr^{-20/3} = -\frac{2ag \sin \phi}{3\pi} + Cr^{-20/3}. \quad (17)$$

But when  $r = b$ ,  $y = 0$ , and, hence,  $C = agb^{20/3} \sin \phi/(30\pi)$ , and (17) becomes

$$\dot{r}^2 = \frac{dr^2}{dt^2} = \frac{ag \sin \phi}{30\pi} \left( \frac{b^{20/3} - r^{20/3}}{r^{20/3}} \right), \quad (18)$$

giving for the required time

$$t = \sqrt{\frac{30\pi}{ag \sin \phi}} \int_0^b \frac{r^{10/3} dr}{\sqrt{b^{20/3} - r^{20/3}}}. \quad (19)$$

The last factor may be put into the form,

$$I = \int_0^b \frac{r^{10/3}}{b^{10/3}} \left( 1 + \frac{r^{20/3}}{2b^{20/3}} + \frac{3}{8} \frac{r^{40/3}}{b^{40/3}} + \dots \right) dr, \quad (20)$$

the series appearing to converge suitably for the degree of approximation for  $t$  in (19) by the conditions of the problem, the result though differing from that stated in the problem.

### 347 (Mechanics). Proposed by E. B. ESCOTT, Kansas City, Mo.

A cord  $ABCD$  is suspended from points  $A$  and  $D$  which are 20 feet apart in horizontal distance.  $D$  is 4 feet lower than  $A$ . At  $B$  and  $C$  are suspended weights 100 and 200 lbs.  $AB = 8$  feet,  $BC = 10$  feet,  $CD = 12$  feet. Find angles  $\alpha$ ,  $\beta$ ,  $\gamma$  made by  $AB$ ,  $BC$ ,  $CD$ , respectively, with the horizontal. Also find the tensions  $T_1$ ,  $T_2$ ,  $T_3$  in  $AB$ ,  $BC$ ,  $CD$ .

### SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

For the equilibrium of  $w_1 = 100$ , resolving vertically and horizontally,

$$T_1 \sin \alpha - T_2 \sin \beta = w_1 = 100,$$

$$T_1 \cos \alpha = T_2 \cos \beta;$$

similarly for  $w_2 = 200$ ,

$$T_3 \sin \gamma + T_2 \sin \beta = w_2 = 200,$$

$$T_3 \cos \gamma = T_2 \cos \beta.$$

The sums of the horizontal and vertical projections of  $AB$ ,  $BC$ ,  $CD$  give

$$8 \cos \alpha + 10 \cos \beta + 12 \cos \gamma = 20,$$

$$8 \sin \alpha + 10 \sin \beta - 12 \sin \gamma = 4.$$

These six equations furnish the theoretical solution of the problem.

The PROPOSER furnished a complete solution.

**348 (Mechanics).** Proposed by **ALTON L. MILLER**, Ann Arbor, Michigan.

If equilateral triangles be constructed on the sides of any triangle, their centers are the vertices of a new equilateral triangle. Show that the center of gravity of this new equilateral triangle coincides with the center of gravity of the original triangle.

**SOLUTION BY EMMA M. GIBSON**, Springfield, Mo.

Let  $ABC$  be the given triangle and let the coördinates of the vertices  $A$ ,  $B$ ,  $C$  referred to the rectangular axes  $ox$  and  $oy$  be  $(c, o)$ ,  $(o, b)$ ,  $(a, o)$ , respectively. The equation of the line through  $(a, o)$  and  $(o, b)$  is

$$y = -\frac{b}{a}x + b. \quad (1)$$

The line from  $D$ , the third vertex of the equilateral triangle on  $BC$ , through  $(a/2, b/2)$  and perpendicular to (1) is

$$y = \frac{a}{b}x + \frac{b^2 - a^2}{2b} \quad (2)$$

The line through  $C$  making an angle of  $60^\circ$  with (1) is

$$y = \frac{b + a\sqrt{3}}{b\sqrt{3} - a}(x - a). \quad (3)$$

Solving equations (2) and (3), the values of the coördinates of  $D$  are found to be  $[(a + b\sqrt{3})/2, (b + a\sqrt{3})/2]$ .

Similarly the coördinates of  $F$  and  $E$  are found to be  $[(a + c)/2, \sqrt{3}(c - a)/2]$  and  $[(c - b\sqrt{3})/2, (b - c\sqrt{3})/2]$ , respectively.

Now the centers  $H$ ,  $G$ ,  $I$  of the three equilateral triangles are

$$\left(\frac{3a + b\sqrt{3}}{6}, \frac{3b + a\sqrt{3}}{6}\right), \quad \left(\frac{a + c}{2}, \frac{\sqrt{3}(c - a)}{6}\right), \quad \left(\frac{3c - b\sqrt{3}}{6}, \frac{3b - c\sqrt{3}}{6}\right),$$

respectively, since  $\bar{x} = \frac{1}{3}(x_1 + x_2 + x_3)$ ,  $\bar{y} = \frac{1}{3}(y_1 + y_2 + y_3)$ . These points are the vertices of the new triangle and by the formula for the length of a line between two points, the three sides are proved equal. Hence, the new triangle is equilateral.

The coördinates of the center of gravity of the original triangle are  $\bar{x} = \frac{1}{3}[a + c]$ ,  $\bar{y} = \frac{1}{3}b$ . The coördinates of the center of gravity of the new triangle are

$$\bar{x} = \frac{1}{3} \left[ \frac{a + b\sqrt{3}}{2} + \frac{a + c}{2} + \frac{c - b\sqrt{3}}{2} \right] = \frac{1}{3}(a + c),$$

$$\bar{y} = \frac{1}{3} \left[ \frac{b + a\sqrt{3}}{2} + \frac{\sqrt{3}(c - a)}{2} + \frac{b - c\sqrt{3}}{2} \right] = \frac{1}{3}b,$$

which are the same as those obtained for the original triangle.

Also solved by **HORACE OLSON** and **ROGER JOHNSON**.

**268 (Number Theory).** Proposed by **FRANK IRWIN**, University of California.

Show that in any arithmetical progression, whose first term  $a_1$  and common difference  $d$  are positive integers, any required number of consecutive terms may be found, no one of which is a prime number.